

TIME TABLE

TIME	Monday May 14	Tuesday May 15	Wednesday May 15	Thursday May 16	Friday May 17
9.00 - 9.45	Registration	Forest	Forest	Bardella	Bertram
9.45 - 10.30	Niordson	Glüge	Forest	Bardella	Bertram
11.00 - 11.45	Müller	Glüge	Bardella	Niordson	Sepecher
11.45 - 12.30	Müller	Glüge	Bardella	Niordson	Sepecher
14.00 - 14.45	Bertram	Niordson	Niordson	Bertram	
14.45 - 15.30	Bertram	Niordson	Müller	Bertram	
16.00 - 16.45	Forest	Bardella	Müller	Sepecher	
16.45 - 17.30	Forest	Bardella	Müller	Sepecher	
18.00	Welcome Aperitif				

ADMISSION AND ACCOMMODATION

The registration fee is 600.00 Euro + VAT*, where applicable (bank charges are not included). The registration fee includes a complimentary bag, four fixed menu buffet lunches (on Friday upon request), hot beverages, downloadable lecture notes and wi-fi internet access.

Applicants must apply at least one month before the beginning of the course. Application forms should be sent on-line through the following web site: <http://www.cism.it>. A message of confirmation will be sent to accepted participants. Applicants requiring assistance with the registration should contact the secretariat at the following email address cism@cism.it.

Applicants may cancel their course registration and receive a full refund by notifying CISM Secretariat in writing (by email to cism@cism.it) no later than two weeks prior to the start of the course.

Cancellation requests received during the two weeks prior to the start of the course will be charged a 50.00 Euro handling fee. Incorrect payments are also subject to a 50.00 Euro handling fee.

A limited number of participants from universities and research centres who are not supported by their own institutions can be offered lodging and/or board, if available, in a reasonably priced hotel or student guest house.

Requests should be sent to CISM Secretariat by **March 14, 2018** along with the applicant's curriculum and a letter of recommendation by the head of the department or a supervisor confirming that the institute cannot provide funding. Preference will be given to applicants from countries that sponsor CISM.

Information about travel and accommodation is available on the web site www.cism.it, or can be mailed upon request.

* Italian VAT is 22%.

For further information please contact:

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MECHANICS OF STRAIN GRADIENT MATERIALS

Advanced School
coordinated by

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Udine May 14 - 18 2018

MECHANICS OF STRAIN GRADIENT MATERIALS

Many materials show a different elastic or plastic behavior if tested on large or on small samples. Such size effects can be ascribed to internal length scales related to the microstructure. This behavior is for instance observed in indenter tests, torsion, bending, and shear banding. Such effects cannot be simulated by classical constitutive models in which the stresses depend locally on the (dimensionless) strains. One possible extension of the classical models relies on the constitutive inclusion of higher strain gradients, which involve the dimension of length. This allows for an introduction of internal length scales in the constitutive model.

Strain gradient models can further be employed for the regularization of singularities in the classical solutions, which make them also advantageous from a numerical point of view. Moreover, they allow for the conceptually sound introduction of line and point forces into continuum mechanics.

The balance laws and boundary conditions for such materials can be derived by resorting to variational principles. For each primal higher strain gradient a conjugate stress tensor of the same tensorial order has to be introduced, for which a constitutive law is needed. Material modeling becomes a challenging task for such materials.

In the elastic case, all stress tensors may depend on all of the corresponding strain tensors. Even in the linear case, this leads to an enormous amount of material constants. This can, however, be drastically reduced by the assumption of symmetry properties like isotropy or centro-symmetry. In the non-linear case of finite deformations, one has to satisfy invariance principles, which is not trivial.

In the case of plastically deforming metals, primal higher strain gradients are chosen with the aim of describing the behavior of geometrically necessary dislocations.

In the course, the following topics will be considered

- experimental findings for size effects
- balance laws and boundary conditions for strain gradient materials
- the linear theory of elasticity and plasticity of strain gradient materials of arbitrary order
- the application to crystal plasticity
- the finite strain gradient theory for large deformations
- the application of strain gradient models to fracture and damage and to micro-to-macro transitions (media with microstructure).

PRELIMINARY SUGGESTED READINGS

Bertram, A., Compendium on Gradient Materials, Otto-von-Guericke-Universität Magdeburg, 240 pages (2017).

Cordero, N.M., Forest, S. and Busso, E.P., Second Strain Gradient Elasticity of Nano-Objects, *J. Mech. Phys. Solids*, 97, 92-124, doi: 10.1016/j.jmps.2015.07.012 (2016).

Liebold, C., Müller, W. H., Comparison of gradient elasticity models for the bending of micromaterials. *Computational Materials Science*, 116, 52-61 (2016).

Gudmundson, P., A unified treatment of strain gradient plasticity. *J. Mech. Phys. Solids*, 52,6, 1379-1406 (2004).

Sepecher, P., Alibert, J.-J., Dell'Isola, F., Linear elastic trusses leading to continua with exotic mechanical interactions. *J. Physics: Conference Series*, 319,1, 13 (2011).

Olive, M., Auffray, N., Symmetry classes for odd-order tensors. *ZAMM - Journal of Applied Mathematics and Mechanics*, 94(5) 421-447 (2014).

Panteghini, A., Bardella, L., On the Finite Element implementation of higher-order gradient plasticity, with focus on theories based on plastic distortion incompatibility, *Computer Methods Appl. Mech. Engrg.* 310, 840-865 (2016).

LECTURES

All lectures will be given in English. Lecture notes can be downloaded from the CISM web site. Instructions will be sent to accepted participants.

INVITED LECTURERS

Lorenzo Bardella - University of Brescia, Italy
6 lectures on: Crystal plasticity.

A strain gradient crystal plasticity theory based on dislocation density tensor is presented, along with its isotropic counterpart, involving the plastic spin. A finite element framework to efficiently simulate both rate-dependent and rate-independent material responses is provided.

Albrecht Bertram - University of Magdeburg, Germany
6 lectures on: Balance laws for gradient materials and finite elasto-plasticity.

Starting with principle of virtual power, the balance laws and the boundary conditions for gradient materials can be derived. A format for a finite gradient elasticity is provided. Reduced forms and symmetry transformations are introduced.

Assisted by **Rainer Glüge** - University of Magdeburg, Germany
with discussion on: Linear gradient elasticity.

Hooke's law is extended to the first strain gradient elasticity. After discussing the underlying principles, we examine the peculiarities that arise in this case. The appearance of odd-order elasticity tensors is a new feature absent in classical media.

Samuel Forest - Mines ParisTech, Evry, France
5 lectures on: From micromorphic to gradient materials.

Starting from Eringen's and Mindlin's higher order theory of materials with microstructure, elastic-plastic constitutive laws are presented at finite deformations. Internal constraints are enforced to recover the strain gradient theory. Applications deal with elasticity of nano-objects and strain localization phenomena in plasticity.

Wolfgang Müller - Technical University of Berlin, Germany
6 lectures on: The experimental evidence of strain gradient elasticity from the micro perspective to macroscopic world including parameter analysis, theoretical basis for an experimental understanding of elastic strain gradient effects, experimental methods used to measure strain gradient elastic constants: Raman spectroscopy, atomic force microscopy, pantograph analysis.

Christian Niordson - Technical University of Denmark, Lyngby, Denmark
6 lectures on: Size-effects and strain gradient plasticity: experiments, theory and modeling.
A general overview of plasticity size-effects in various experiments is given like indentation, torsion, and thin film deformation. Strain gradient plasticity theory is introduced together with numerical solution techniques.

Pierre Seppecher - University of Toulon, France
4 lectures on: Pantographs and graph-based structures.
Materials with pantographic microstructure and their generalization to graph-based structures are examples of materials with a clear microscopic interpretation of strain gradient effects. They also give an enlightening illustration of the concept of double forces.